MITES 2010 ADVANCED CALCULUS
FORMULA SHEET FOR THE FINAL

THURSDAY, JULY 29TH

$$\sin 2x = 2\sin x\cos x \qquad \cos^2 x = \tfrac{1}{2}(1+\cos 2x) \qquad \sin^2 x + \cos^2 x = 1$$
$$\cos 2x = \cos^2 x - \sin^2 x \qquad \sin^2 x = \tfrac{1}{2}(1-\cos 2x) \qquad \tan^2 x = \sec^2 x - 1$$

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arc length for a curve $y = f(x)$: $\int_a^b \sqrt{1+f'(x)^2}\,dx$

arc length for a parametric curve $c(t) = (x(t), y(t))$: $\int_a^b \sqrt{x'(t)^2 + y'(t)^2}\,dt$

arc length for a polar curve $r = f(\theta)$: $\int_\alpha^\beta \sqrt{f(\theta)^2 + f'(\theta)^2}\,d\theta$

area for a polar curve $r = f(\theta)$: $\int_\alpha^\beta \frac{1}{2} f(\theta)^2\,d\theta$

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dot product: $\mathbf{a}\cdot\mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3 = \|\mathbf{a}\|\|\mathbf{b}\|\cos\theta$

cross product: $\mathbf{a}\times\mathbf{b} = (a_2b_3 - a_3b_2, a_3b_1 - a_1b_3, a_1b_2 - a_2b_1)$

$\|\mathbf{a}\times\mathbf{b}\| = \|\mathbf{a}\|\|\mathbf{b}\||\sin\theta|$

equation of a plane: $(\mathbf{x} - \mathbf{x_0})\cdot\mathbf{n} = 0$

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discriminant: $D = \frac{\partial^2 f}{\partial^2 x}\frac{\partial^2 f}{\partial^2 y} - \left(\frac{\partial^2 f}{\partial x \partial y}\right)^2$

second derivative test:

- if $D > 0$ and $\frac{\partial^2 f}{\partial^2 x} > 0$, then local minimum
- if $D > 0$ and $\frac{\partial^2 f}{\partial^2 x} < 0$, then local maximum
- if $D < 0$, then saddle point
- if $D = 0$, then the test is inconclusive

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plane polars:	$(r\cos\theta, r\sin\theta)$	$dx\ dy \to r\ dr\ d\theta$
cylindrical polars:	$(r\cos\theta, r\sin\theta, z)$	$dx\ dy\ dz \to r\ dr\ d\theta\ dz$
spherical polars:	$(r\cos\theta\sin\phi, r\sin\theta\sin\phi, r\cos\phi)$	$dx\ dy\ dz \to r^2\sin\phi\ dr\ d\theta\ d\phi$

Green's Theorem:	$\int\int_{\mathcal{D}} \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\ dx\ dy = \oint_{\partial\mathcal{D}} P\ dx + Q\ dy$
Divergence Theorem:	$\int\int\int_{\mathcal{V}} \nabla\cdot\mathbf{F}\ dV = \int\int_{\mathcal{S}} \mathbf{F}\cdot d\mathbf{S}$
Stokes's Theorem:	$\int\int_{\mathcal{S}} \nabla\times\mathbf{F}\cdot d\mathbf{S} = \oint_{\partial\mathcal{S}} \mathbf{F}\cdot d\mathbf{s}$
