MITES 2010 ADVANCED CALCULUS
PROBLEM SET 5

DUE SUNDAY, JULY 25TH

1. Verify the Divergence Theorem in the case of the vector field $\mathbf{F}(x, y, z) = (x, 2, z^2)$ taken over the region bounded by the planes $z = 0$, $z = 4$, $x = 0$, $y = 0$ and the surface $x^2 + y^2 = 4$ in the first octant. Hint: It may help to use cylindrical polar coordinates to evaluate one of the surface integrals.

2. Using the Divergence Theorem, determine $\int_S \mathbf{F} \cdot d\mathbf{S}$ where the vector field $\mathbf{F}$ is given by $\mathbf{F}(x, y, z) = (x, xy, 2)$ taken over the region bounded by the planes $z = 0$, $z = 4$, $x = 0$, $y = 0$ and the surface $x^2 + y^2 = 9$ in the first octant.

3. A surface consists of five sections formed by the planes $x = 0$, $x = 1$, $y = 0$, $y = 3$, $z = 2$ in the first octant. Under the assumption that the vector field $\mathbf{F}(x, y, z) = (y, z^2, xy)$ exists over the surface and around its boundary, verify Stokes' Theorem.

4. A surface S consists of that part of the cylinder $x^2 + y^2 = 9$ between $z = 0$ and $z = 4$ for $y \geq 0$ and the two semicircles of radius 3 in the planes $z = 0$ and $z = 4$. If $\mathbf{F}(x, y, z) = (z, xy, xz)$, evaluate $\int_S \nabla \times \mathbf{F} \cdot d\mathbf{S}$ using Stokes's Theorem.

5. Explain the terms solenoidal, irrotational and conservative vector field, and give an example in each case.

6. Briefly state and describe Maxwell's Equations. Show, by taking the *curl* of Ampère's Law, that the magnetic field satisfies the wave equation $\nabla^2 \mathbf{B} = \frac{\partial^2 \mathbf{B}}{\partial t^2}$.

7. Use cylindrical polar coordinates to evaluate the integral $\int\int\int_V \frac{z}{\sqrt{x^2+y^2}}\, dx\, dy\, dz$, where V is the solid bounded above by the plane $z = 2$ and below by the surface $x^2 + y^2 - 2z = 0$.

8. Use spherical polar coordinates to evaluate the integral $\int\int\int_V \frac{dx\, dy\, dz}{\sqrt{x^2+y^2+z^2}}$ over the solid hemisphere $x^2 + y^2 + z^2 \leq 3$ with $z \geq 0$.

Please send any comments or corrections to julia.wolf@cantab.net.