

**MITES 2010 ADVANCED CALCULUS  
REVIEW SHEET FOR MIDTERM**

1. Find the area enclosed by the curve  $r = 5 \sin \theta$  between  $\theta = 0$  and  $\theta = \pi/3$ .
2. Find the length of the polar curve  $r = 4 \sin^2 \frac{\theta}{2}$  between  $\theta = 0$  and  $\theta = \pi$ .
3. Prove that (i)  $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = \mathbf{B} \cdot (\mathbf{C} \times \mathbf{A})$ ; (ii)  $\mathbf{B} \cdot (\mathbf{A} \times \mathbf{C}) = -\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C})$ ; (iii)  $\mathbf{C} \cdot (\mathbf{A} \times \mathbf{C}) = 0$ .
4. Show that  $(\mathbf{a} \times \mathbf{b}) \times (\mathbf{a} \times \mathbf{c}) = (\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}))\mathbf{a}$ .
5. Use the cross product to find the area of the triangle spanned by the three points  $(2, 1, 0)$ ,  $(3, 1, 5)$  and  $(0, 2, -1)$ .
6. If  $V = x^2 + y^2 + z^2$ , express  $x \frac{\partial V}{\partial x} + y \frac{\partial V}{\partial y} + z \frac{\partial V}{\partial z}$  in its simplest form.
7. If  $V = 250$  volts and  $R = 50$  ohms, find the approximate change in  $I$  resulting from an increase of 1 volt in  $V$  and an increase of 0.5 ohm in  $R$ .
8. If  $z = 2xy - 3x^2y$  and  $x$  is increasing at  $2m/s$ , determine the rate at which  $y$  must be changing so that at the instant when  $x = 3m$  and  $y = 1m$ ,  $z$  is neither increasing nor decreasing.
9. In what direction should one travel from the origin to obtain the most rapid decrease of the function  $f(x, y, z) = (2 - x - y)^2 + (3x + 2y - z + 1)^2$ ?
10. Use the chain rule to evaluate  $\frac{\partial f}{\partial s}$  for  $f(x, y, z) = x^3y^2 - 2z$  with  $x = 2s^2t$ ,  $y = st^3$  and  $z = st^2$ .
11. Find  $\frac{\partial z}{\partial x}$  when  $x^2y + y^2z + xz^2 = 3$ .
12. If  $z = e^x(x \cos y - y \sin y)$ , show that  $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$ .
13. Find the directional derivative of  $f(x, y, z) = 4xy + z^3$  at  $(3, -1, 1)$  in the direction pointing to the origin.
14. Find a function  $f$  such that  $\nabla f = (z, 2y, x)$ .
15. Find an equation of the tangent plane to the surface  $x^2 + 3y^2 + 4z^2 = 20$  at  $(2, 2, 1)$ .

16. Determine the critical points of the function  $f(x, y) = (x^2 + y^2)e^{-x}$ , and use the Second Derivative Test to establish whether they are local maxima, minima or saddle points.

17. Find the global extrema of the function  $f(x, y) = x^3 + x^2y + 2y^2$  on the domain  $x, y \geq 0$ ,  $x + y \leq 1$ .

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