1. What does it mean for a set $A$ to be countable? Is the set of all rationals with prime denominator countable? Justify your answer.

2. What does it mean for the cardinality of a set $X$ to be greater than or equal to the cardinality of a set $Y$? What is the relationship between the cardinality of $\mathbb{N}$ and the cardinality of the set of functions $f : \mathbb{N} \to \{0, 1\}$?

3. Exhibit a bijection that shows that $(0, 1]$ is a continuum.

4. True or false? If $(a_n)_{n \in \mathbb{N}}$ has a bounded subsequence, then $(a_n)_{n \in \mathbb{N}}$ has a convergent subsequence. Justify your answer.

5. Give two equivalent definitions of what it means for an element $a \in \mathbb{R}$ to be an accumulation point of the sequence $(a_n)_{n \in \mathbb{N}}$. 
6. What is the \( \lim \inf_{n \to \infty} \) of the sequence \( (a_n)_{n \in \mathbb{N}} \) defined by \( a_{2n} = 2n, \ a_{2n+1} = 1 + \frac{1}{n} \) for all \( n \in \mathbb{N} \)?

7. Let \( (a_n)_{n \in \mathbb{N}} \) be defined by \( a_n = (-1)^n \) for all \( n \in \mathbb{N} \). Show that \( (a_n)_{n \in \mathbb{N}} \) is not a Cauchy sequence, and deduce that \( (a_n)_{n \in \mathbb{N}} \) does not converge.

8. Define what it means for a function to be uniformly continuous on its domain, and state Cantor’s theorem on uniform continuity.

9. Is the sequence of functions \( f_n : [0, \pi] \to \mathbb{R} \) defined by \( f_n(x) = \sin^n(x) \) uniformly convergent? Justify your answer.