Further Topics in Analysis: Sample Exam Question

B. Let $A \subseteq \mathbb{R}$ and $(f_n(x))_{n \in \mathbb{N}}$ be a sequence of functions from $A$ to $\mathbb{R}$.

(a) (6 marks)
   i) What does it mean to say that the sequence $(f_n(x))_{n \in \mathbb{N}}$ converges pointwise on $A$ to a function $f : A \to \mathbb{R}$?
   ii) What does it mean to say that the sequence $(f_n(x))_{n \in \mathbb{N}}$ converges uniformly on $A$ to a function $f : A \to \mathbb{R}$?

(b) (4 marks)
   State without proof Weierstrass’s Theorem on Uniform Convergence.

(c) (8 marks)
   Find the pointwise limit of the sequence $f_n(x) = x^n$ ($n \in \mathbb{N}$) on the closed segment $[0, 1]$. Is this convergence uniform? Justify your answer.

(d) (12 marks)
   Find the pointwise limit of the sequence $f_n(x) = \frac{e^x}{n}$ ($n \in \mathbb{N}$) on $\mathbb{R}$. Is this convergence uniform? Justify your answer.
   Now restrict the domains of $f_n$ to $[0, 100]$. Is the convergence uniform now? Justify your answer.

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