

**MITES 2010 ADVANCED CALCULUS
WARM-UP PROBLEMS**

DUE TUESDAY, JUNE 22

1. Find a point on the curve

$$y = 2x^3 - 4x + 1$$

whose tangent line is parallel to the line $y - 2x = 4$. If there is more than one such point, determine all points with this property, and give the equations of the respective tangents.

2. Without the help of your calculator, sketch the graph of

$$f(x) = \frac{x^2 - 3}{x - 2}, x \neq 2,$$

including all its asymptotes.

3. Sketch the area enclosed by the curve $y = x^2$, the lines $x = 1$, $x = 2$ and the x -axis. Visualize and find the volume of the solid obtained by rotating this area about the axis $y = -1$.

4. Suppose that $f(2) = 3$ and $f'(2) = 2$. Find

$$\frac{d}{dx} \left[\frac{f(x)}{x^2} \right]$$

at $x = 2$.

5. Evaluate the following indefinite integrals:

(1)

$$\int x^3 e^x dx$$

(2)

$$\int \cos^3 x \sin^2 x dx$$

6. In your own words, describe what it means for a function $f : \mathbb{R} \rightarrow \mathbb{R}$ to be continuous, and what it means for this function to be differentiable. Give an example of a function which is continuous but not differentiable at $x = 5$.

Please send any comments or corrections to julia.wolf@cantab.net.